LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

PMT3MC01 – TOPOLOGY

Date: 07-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	Define discrete topology with an example.
b)	Can you say that union of two topologies on a set *X* a topology on *X*? Justify
c)	What do you mean by a homeomorphism between two topological spaces?
d)	When can you say that a topological space is separated?
e)	Define a normal space.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	If $X=\{a,b,c\}$ and for the given topologies $I_1=\{X,\varnothing,\{a\}\}$ and $I_2=\{X,\varnothing,\{b\}\}$ then (i) I_1 is finer than I_2. (ii) I_2 is finer than I_1. (iii) I_1 and I_2 are not comparable. (iv) None of the above.
b)	$Y=[0,1]\cup(2,3)$ in *R* (i) $[0,1]$ is only closed but not open in *Y*. (ii) $(2,3)$ is only open but not closed in *Y*. (iii) $[0,1]$ and $(2,3)$ are neither open nor closed in *Y*. (iv) $[0,1]$ and $(2,3)$ are both open and closed in *Y*.
c)	A topological space *X* is connected if and only if (i) the only sets both open and closed are $\varnothing$ and *X*. (ii) $X=A\cup B$ where *A* and *B* are disjoint, nonempty open subsets of *X*. (iii) *X* is path connected. (iv) None of the above.
d)	In the real line *R* which of the following is true? (i) (0,1]is compact (ii) [0,1]is compact (iii) (0,1]is closed (iv) (0,1]is not bounded
e)	In a topological space, which of the following statement is true? (i)Any compact subset of a Hausdorff space need not be closed. (ii) Any closed subset of a compact space need not be compact. (iii) Any limit point compact space is compact space.

	(iv) Any limit point compact space is sequentially compact.

	SECTION B – K3 (CO2)
	Answer any THREE of the following (3 x 10 = 30)
3	Prove the sequence lemma and prove that a function $f: X \to Y$, where *X* is metrizable space and *Y* is a topological space is continuous if and only if for every convergent sequence $x_n \to x$ in *X*, the sequence $f(x_n)$ converges to $f(x)$.
4	Demonstrate three equivalent conditions on the continuity of a function *f* from a topological space *X* to a topological space *Y*.
5	Construct a metric space $(X, \overline{d})$ from the given metric space (X, d) such that every subset of *X* is bounded with respect to metric $\overline{d}$.
6	Sketch the proof of any interval or a ray in a linear continuum *L* in the order topology is connected.
7	Is every closed interval in *R* uncountable? Justify it.
	SECTION C – K4 (CO3)
	Answer any TWO of the following (2 x 12.5 = 25)
8	Is the union of connected sets having a point in common connected? Justify.
9	Analyze the following statement: "If $f: X \to Y$ is a bijective continuous function, *X* is compact and *Y* is Hausdorff, then *f* is a homeomorphism"
10	Examine whether the following statements are true? (i) *X* is compact metrizable space implies *X* is limit point compact space. (ii) *X* is sequentially compact metrizable space implies *X* is compact space.
11	What are the separation axioms? Formulate them.
	SECTION D – K5 (CO4)
	Answer any ONE of the following (1 x 15 = 15)
12	Compare the topologies on R^n induced by the Euclidean metric *d* and the square metric *ρ* and the product topology on R^n.
13	Is the product of finitely compact spaces compact? Justify it with a supportive proof.
	SECTION E – K6 (CO5)
	Answer any ONE of the following (1 x 20 = 20)
14	Design a characterization of a subset *A* of R^n is compact.
15	Demonstrate Urysohn lemma.

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